B.Sc. (Honours) Examination, 2019
Semester-III (CBCS)
Statistics
Course : CC-6
(Statistical Inference)

Time : 3 Hours **Full Marks : 40**

Questions are of value as indicated in the margin.
Answer **any four** of the following questions

1. a) State the desirable properties of an estimator. When do we call a statistic is sufficient for an unknown parameter? 5
   b) Find sufficient statistic for the parameter θ of U(0, θ) distribution. 5
2. a) Define a consistent estimator. Show that if T_n is a consistent estimator of θ and γ(θ) is a continuous function of θ, then γ (T_n) is a consistent estimator of γ(θ). 6
   b) If $x_1, x_2, \ldots x_n$ are random observations on a Bernoulli variate X taking the value 1 with probability p and the value 0 with probability (1-p) then show that $\frac{\Sigma x_i}{n}\left(1-\frac{\Sigma x_i}{n}\right)$ is a consistent estimator of p(1-p). 4
3. a) Let $x_1, x_2, \ldots, x_n$ be random sample from the distribution with mass function
$$f(x,\theta)=\theta^x(1-\theta)^{1-x}, x=0,1; 0<\theta<1$$
   Examine whether $T=\sum_{i=1}^{n} X_i$ is complete for this distribution or not. 5
   b) Show that in case of Pareto distribution
$$f(x,\alpha)=\frac{\alpha}{x^{\alpha+1}}, x\geq 1, \alpha>0$$
   The method of moments fails if $0<\alpha<1$. Derive the method of moment estimator when $\alpha>1$. 5
4. a) Obtain the maximum likelihood estimator of θ in
$$f(x,\theta)=(1+\theta)x^{\theta}; 0<x<1$$
   based on a sample of size *n*. 4
   b) Develop the sequential probability Ratio Test (SPRT) of strength (α,β) to test the hypothesis
$$H_0 : \theta = \theta_0$$
   against
$$H_1 : \theta = \theta_1$$
   for the distribution
$$f(x,\theta)=\theta^x(1-\theta)^{1-x}, \quad \begin{matrix} x=0,1 \\ 0<\theta<1 \end{matrix}$$
   6
5. a) Bring out the difference between a randomized test and a non-randomized test. Explain how the decision based on a randomized test can be taken in the discrete set-up. 4

P.T.O.

b) Let $x_1, x_2, \ldots x_n$ be a random sample from $N(\theta, \sigma^2)$ σ^2 is not specified. Derive size α likelihood ratio test for testing

$$H_0 : \theta = \theta_0$$

against

$$H_1 : \theta = \theta_0$$ 6

6. a) If $x_1, x_2, \ldots, x_n$ is a random sample from the distribution with density function

$$f(x,\theta) = \begin{cases} \theta x^{\theta-1} & \theta < x < 1 \\ 0 & \text{Otherwise} \end{cases}$$

where $0 < \theta < \infty$. Show that the MP test of level α for testing $H_0 : \theta = 1$ against $H_1 : \theta = 2$ is given by the critical region.

$$\left\{ \underset{\sim}{x} \mid \prod_{i=1}^{n} x_i > e^{-\frac{1}{2}\chi^2_{1-\alpha,2n}} \right\}$$

where $\chi^2_{1-\alpha,2n}$ is the lower α-point of the χ^2 distribution with 2n degrees of freedom.5

b) It is required to test H_0 against H_1 from a single observation x, where H_0 is the hypothesis that the pdf is

$$f(x) = \frac{1}{\sqrt{2n}} e^{-\frac{1}{2}x^2}, -\infty < x < \infty$$

and H_1 is the hypothesis that the pdf is

$$f(x) = \frac{2}{\left\lceil \frac{1}{4} \right.} e^{-x^4}, -\infty < x < \infty$$

Obtain the most powerful (MP) test with level of significance α in this case. 5
